Elementary Mathematics for Teachers

Homework for Standards Edition

This booklet contains homework for *Elementary Mathematics for Teachers* (EMT) for use with the Standards Edition of the *Primary Mathematics* textbooks

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Adapted to the *Primary Mathematics* Standards Edition by Benjamin Ellison and Daniel McGinn.

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How to use these notes

The textbook *Elementary Mathematics for Teachers* is designed to be used in conjunction with the Primary Mathematics (U.S. Edition) textbooks, and many of the homework exercises refer to specific pages in these books. After the 2008 publication of the Standards Edition of the Primary Mathematics books some readers, most notably in-service teachers in California, have had ready access to the Standards Edition, but not to the U.S. Edition books. This reference booklet delineates the changes to the text and the homework assignments needed for readers who prefer to use the Standards Edition instead of the U.S. Edition.

You will need *Elementary Mathematics for Teachers* (EMT) and the following "Standards Edition" Primary Mathematics textbooks:

- Primary Math Standards Edition Textbooks 2A and 2B (referenced in text, not needed for exercises).
- Primary Math Standards Edition Textbooks 3A and 3B.
- Primary Math Standards Edition Textbook 4A.
- Primary Math Standards Edition Textbooks 5A and 5B, and Workbook 5A.

There are major differences between the editions of the grade 6 textbooks that make it impractical to convert the grade 6 textbook references in EMT to the Standards Edition. Consequently, you will also need a copy of:

- Primary Math **US Edition** Textbook 6A.

All of these textbooks can be ordered from the website SingaporeMath.com.

The format of this booklet is straightforward. Each section of *Elementary Mathematics for Teachers* is listed. Any needed changes to the text of EMT for that section are listed immediately after the section name. For about half of the sections, the homework set in this booklet completely replaces the one in EMT. The remaining sections, such as Section 2.1 below, require no changes at all, so the homework problems can be done directly from EMT. **In this booklet, unless otherwise noted, all page number references to the Primary Math textbooks grades 1-5 refer to the Standards Edition, while all references to Primary Math 6A refer to the U.S. Edition**.

The authors thank Ben Ellison and Dan McGinn for meticulously checking each homework problem and replacing the page number references from the U.S. edition with references to the Standards edition. We also thank the staff at Sefton-Ash Publishing, and Dawn and Jeffery Thomas at SingaporeMath.com for their support in the creation and distribution of this booklet.

Finally, a note for instructors. The revisions here are intended for the convenience of teachers whose schools use the Standards Edition. In college courses for pre-service teachers, and in general professional development settings, the U.S. Edition of the *Primary Mathematics* series remains the recommended series for use with *Elementary Mathematics for Teachers*.

Scott Baldridge

Thomas Parker

May, 2010

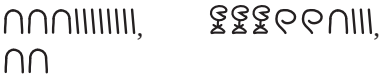
CHAPTER 1

Place Value and Models for Arithmetic

1.1 Counting

Homework Set 1

1. Make the indicated conversions.

   a) To decimal:  , , ,

   b) To Egyptian: 8, 37, 648, 1348

   c) To decimal: MMMDCCXXXIII, MCMLXX, MMLIX, CDXLIV

   d) To Roman: 86, 149, 284, 3942

2. Write the number 8247 as an Egyptian numeral. How many fewer symbols are used when this number is written as a decimal numeral?

3. a) Do column addition for the Egyptian numerals below. Then check your answer by converting to decimal numerals (fill in and do the addition on the right).

   ↔ 135 + _____

   Write a similar pair of column additions for

   b) 273 + 125 and

   c) 328 + 134.

   d) Write a sentence explaining what you did with the 12 tallies that appeared in the sum c) in Egyptian numerals.

4. Make up a first-grade word problem for the addition 7 + 5 using a) the set model and another b) using the measurement model.

5. Open Primary Math 3A to page 11 and read Problem 3, and then read Problem 2 on page 25. Then write the following as decimal numerals.

   a) 6 billion 3 thousand 4 hundred and 8

   b) 2 quadrillion 3 billion 9 thousand 5 hundred 6

   c) 230 hundreds 32 tens and 6 ones

   d) 54 thousands and 26 ones

   e) 132 hundreds and 5 ones.

6. Write the following numerals in words.

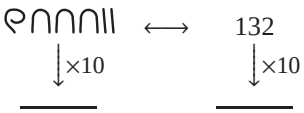
   a) 1347 b) 5900
   c) 7058 d) 7,000,000,000
   e) 67,345,892,868,736

7. Multiply the following Egyptian numerals by ten *without converting or even thinking about decimal numbers*.

   a) b) c)

   d) e) f)

8. a) Fill in the missing two corners of this chart.



b) Fill in the blanks: to multiply an Egyptian numeral by 10 one shifts symbols according to the rule | → ___ → ___ → ___.

c) To multiply a decimal numeral by 10 one shifts . . . what?

1.2 The Place Value Process

Corrections to text: Change page 10, line -18 to read: "The place value skills that are developed in Primary Math 3A pages 8-17 are extended to larger numbers in Primary Math 4A pages 8-19, and further extended in Primary Math 5A pages 8-10 . . . ".

Change page 10, line -8 to read: "This is nicely illustrated in Primary Math 3A, page 13, Problem 9 — have a look. The problems on pages 13 and 14 . . . "

Homework Set 2

Study the Textbook! *In many countries teachers study the textbooks, using them to gain insight into how mathematics is developed in the classroom. We will be doing that daily with the Primary Mathematics textbooks. The problems below will help you study the beginning pages of Primary Math 3A, 4A and 5A.*

1. Primary Math 3A begins with 10 pages (pages 8–17) on place value. This is a review. Place value ideas were covered in grades 1 and 2 for numbers up to 1000; here those ideas are extended to 4–digit numbers. Many different ways of thinking about place value appear in this section.
   a) Read pages 8–12 carefully. These help establish place value concepts, including chip models and the _____ form of 4–digit numbers.
   b) The problems on page 13 use chip models and expanded form to explain some ideas about putting numbers in order. The picture at the top of page 13 helps students see that to compare 316 and 264 one should focus on the digit in the ____ place.
   c) The illustrations comparing 325 and 352 show that when the first digits are the same, the ordering is determined by what? Why did the authors choose numbers with the same digits in different orders?
   d) Parts (a) and (b) of Problem 9 ask students to compare 4–digit numbers. What place value must be compared for each of these four pairs of numbers?
   e) What digit appears for the first time in Problem 10a?
   f) Solve Problems 11, 13, and 14 on page 14.
   g) On the same page, list the two numbers which answer Problem 12, then the two numbers which answer Problem 15.
2. a) Continuing in Primary Math 3A, explain the strategy for solving Problem 16a on page 14.
   b) What is the smallest 4-digit number you can make using all of the digits 0, 7, 2, 8?
   c) Do Problem 5 on page 12. (This Problem does not appear in the Standards edition. It is reproduced below.) This is a magnificent assortment of place-value problems! Write the answers as a list: 1736, 7504, . . ., omitting the labels (i), (ii), (iii), etc. We will refer to this way of writing answers as *list format*.

      5. Find the missing numbers:
         i. 1000 + 700 + 30 + 6 = □
         ii. 7000 + 500 + 4 = □
         iii. 3000 + □ = 3090
         iv. 6000 + □ + 2 = 6802
         v. 4243 = 4000 + 200 + 40 + □
         vi. 4907 – □ = 4007
   d) Do Problems 7–9 on pages 25–26, answering each of the problems in list format. Note that Problems 8 and 9 again use numbers with the same digits in different orders, forcing students to think about place value.
   e) Read pages 15–17, answering the problems mentally as you read. These show students that it is easy to add 10, 100, or 1000 to a number. On page 16, the top chip model shows that to add 100 one needs only to think about the digit in the hundreds place. The bottom chip model shows that to _____ one need only think about the _____ digit.
3. Read pages 8–19 in Primary Math 4A, doing the problems mentally as you read.

a) Do Problem 25 on page 17 in your Primary Math book (do not copy the problem into your homework). The problem is self-checking, which gives the student feedback and saves work for _____.

b) Do Problem 11 on page 13 and Problem 15 on page 15. These extend place value cards and chip models to 5–digit numbers. Fourth-graders are ready for large numbers!

c) Do Problem 13 on page 14. Part (a) asks for a chip model as in Problem 12. This problem shows one 'real life' use of large numbers.

d) Answer Problem 28a on page 18 in list format. This asks students to identify *unmarked* points on the number line.

e) Do Problem 33 mentally. The thinking used to solve part (a) can be displayed by writing: 6000 + 8000 = (6 + 8) thousands = 14, 000. Write similar solutions for parts (c), (d), and (f).

4. Read pages 8–19 of Primary Math 4A, answering the problems mentally as you read. Write down solutions to Problems 1ach, 2e, 3e, 4cf on pages 20 and 21, 27ab on page 17, and 32c on page 19.

5. a) Study page 23 of Primary Math 5A. Write the answers to Problems 1, 3, and 5 on page 24 in list format.

b) Study page 25 and Problem 2 on page 26. Write the answers to Problems 1 and 3 on page 26 in list format.

6. In decimal numerals the place values correspond to powers of ten (1, 10, 100, 1000...). If one instead uses the powers of five (1, 5, 25, 125, ...) one gets what are called 'base 5 numerals'. The base 5 numeral with digits 2 4 3, which we write as $(243)_5$ for clarity, represents 2 twenty-fives + 4 fives +3 ones=73. To express numbers as base 5 numerals, think of making change with pennies, nickels, quarters, and 125¢ coins; for example 47 cents = 1 quarter+4 nickels+2 pennies, so $47 = (142)_5$.

a) Convert $(324)_5$ and $(1440)_5$ to decimal numerals.

b) Convert 86 and 293 to base 5 numerals.

c) Find $(423)_5 + (123)_5$ by adding in base 5. (Think of separately adding pennies, nickels, etc., rebundling whenever a digit exceeds 4. Do not convert to decimal numerals).

1.3 Addition

Homework Set 3

1. Illustrate the equality 3 + 7 = 7 + 3 using (a) a set model, and (b) a bar diagram.

2. Which thinking strategy or arithmetic property (or properties) is being used?

a) 86 + 34 = 100 + 20
b) 13, 345 + 17, 304 = 17, 304 + 13, 345
c) 0 + 0 = 0
d) 34 + (82 + 66) = 100 + 82
e) 2 thousands and 2 ones is equal to 2 ones and 2 thousands.

3. *(Mental Math)* Find the sum mentally by looking for pairs which add to a multiple of 10 or 100, such as 91 + 9 = 100 in Problem a).

a) 91 + 15 + 9
b) 4 + 17 + 32 + 23 + 36 + 20
c) 75 + 13 + 4 + 25
d) 11 + 45 + 34 + 55. e) 34 + 17 + 6 + 23
f) 28 + 32 + 35 + 7.

4. One can add numbers which differ by 2 by a "relate to doubles" strategy: take the average and double. For example, 6 + 8 by twice 7. Use that strategy to find the following sums.

a) 7 + 9 b) 19 + 21
c) 24 + 26 d) 6 + 4.

5. *(Mental Math)* Do Problem 21a on page 33 of Primary Math 3A using compensation.

6. *(Thinking Strategies)* Only a few of the 121 "Addition within 20" facts need to be memorized through practice. Learning to add 1 and 2 by counting-on leaves 99 sums to learn. Adding 0 or 10 is easy, and using compensation to add 9 reduces the list further. After learning to use commutativity, students are left with only 21 facts:

3+3	3+4	3+5	3+6	3+7	3+8
	4+4	4+5	4+6	4+7	4+8
		5+5	5+6	5+7	5+8
			6+6	6+7	6+8
				7+7	7+8
					8+8

Make a copy of this table and answer the following questions.

a) In your table, circle the doubles and tens combinations (which students must learn). How many did you circle?

b) Once they know doubles, students can add numbers which differ by 1 (such as 3 + 4) by relating to doubles — no memorization required. Cross out all such pairs in your table. How many did you cross out?

c) How many addition facts are left? How many additions within 20 require memorization?

7. Match the symbols $=, \approx, \leq, \neq, \geq$ to the corresponding phrase.

a) is less than or equal to

b) is equal to

c) is greater than or equal to

d) is approximately equal to

e) is not equal to

8. Here are some common examples of inappropriate or incorrect uses of the symbol "=".

a) A student writes "Ryan= $2". What should he have written?

b) A student answers the question "Write 4.8203 correct to one decimal place" by writing 4.8203 = 4.8. What should she have written?

c) A student answers the question "Simplify $(3 + 15) \div 2 + 6$" by writing $3 + 15 = 18 \div 2 = 9 + 6 = 15$. What should he have written?

1.4 Subtraction

Homework Set 4

1. a) Illustrate 13 – 8 by crossing out objects in a set model.
   b) Illustrate 16 – 7 on the number line.
2. (*Study the Textbook!*) Study pages 34–40 of Primary Math 3A and answer the following questions.
   a) How does the pictured "student helper" define the difference of two numbers? What is the difference between 9 and 3? The difference between 3 and 9? Do you see how this definition avoids negative numbers?
   b) State which interpretation is used in the following subtraction problems: (i) The questions on page 45 and Problem 3 (ii) Page 49, Problems 5, 6b, and 7a.
3. *(Mental Math)* Do the indicated calculations mentally by looking for pairs whose difference is a multiple of 10.
   a) 34 + 17 – 24 – 27 b) 28 – 16 + 36 – 4.
4. *(Mental Math)* Do the indicated subtractions mentally by "counting up".
   a) 14 – 8 b) 178 – 96 c) 425 – 292.
5. *(Mental Math)* Do the indicated calculations mentally using compensation.
   a) 57 – 19 b) 86 – 18
   c) 95 – 47 d) 173 – 129.
6. a) Illustrate the take-away interpretation for 54 – 28 using a set model. (Draw pennies and dimes and cross some out, but be careful!)
   b) Illustrate the counting-up method for finding 54 – 28 by showing two hops on the number line.
   c) Illustrate the comparison interpretation for 54 – 28 using a set model (use pennies and dimes again and ask a question).
   d) Illustrate the comparison interpretation for 54 – 28 using a measurement model. (Before you start, examine *all* the diagrams in this section).
7. Make up first grade word problems of the following types:
   a) The take-away interpretation for finding 15 – 7.
   b) The part-whole interpretation for 26 – 4.
   c) The comparison interpretation for 17 – 5.
8. Answer the following questions about this section:
   a) In which grade should teaching of subtraction facts begin?
   b) What is "subtraction within 20"?

1.5 Multiplication

Corrections to text: Change page 29, line 11 to read: "This is done in Primary Math 3A pages 108–139."

Homework Set 5

1. (*Study the Textbook!*) The following questions will help you study Primary Math 3A.

   a) Problem 4 on page 70 shows multiplication as a rectangular array and as repeated addition, in order to illustrate the _____ property of multiplication.

   b) (i) Read Problems 5 and 6 on page 71 and Problem 16 on page 75. For each, identify which model for multiplication is being used. (ii) Problems 3 and 4 on page 79 describe set model situations, but illustrate them using _____. (iii) The word problems on page 80 use a variety of models. Which is used in Problem 6? In Problem 9? (iv) Which model is used in the three illustrated problems on page 82?

   c) What is the purpose of four-fact families such as those in Problem 11 on page 73?

2. Continuing in Primary Math 3A,

   a) What are students asked to make on pages 108 –109? What will they be used for?

   b) On page 111, what model is used in Problem 1? What property is being illustrated in 1b?

   c) Problem 2 on pages 111–112 shows how one can use a known fact, such as $6 \times 5 = 30$, to find related facts, such as 6×6 and 6×7. What arithmetic property is being used?

   d) Draw a rectangular array illustrating how the fact $6 \times 6 = 36$ can be used to find 6×12.

   e) Problem 3 on page 113 shows that if you know the multiplication facts obtained from skip counting by 6 then you know ten additional facts by the _____ property.

3. Illustrate the following multiplication statements using a set or rectangular array model:

   a) $5 \times 3 = 3 \times 5$
   b) $2 \times (3 \times 4) = 6 \times 4$
   c) $3(4 + 5) = (3 \times 4) + (3 \times 5)$
   d) $6 \times 1 = 6$.

4. Identify the arithmetic property being used.

   a) $7 \times 5 = 5 \times 7$
   b) $6 + 0 = 6$
   c) $3 + (5 + 2) = (3 + 5) + 2$
   d) $\mid \times \cap\cap\mid\mid\mid = \cap\cap\mid\mid\mid$
   e) $3 + 4 = 1(3 + 4)$
   f) $3(8 \times 6) = (3 \times 8)6$
   g) $(7 \times 5) + (2 \times 5) = (7 + 2) \times 5$

5. *(Mental Math)* Multiplying a number by 5 is easy: take half the number and multiply by 10. (For an odd number like 17 one can find 16×5 and add 5.) Use that method to mentally multiply the following numbers by 5: 6, 8, 7, 12, 23, 84, 321. Write down your answer in the manner described in the box at the end of Section 2.1.

6. *(Mental Math)* Compute 24×15 in your head by thinking of 15 as $10 + 5$.

7. *(Mental Math)* Multiplying a number by 9 is easy: take 10 times the number and subtract the number. For example, $6 \times 9 = 60 - 6$ ("6 tens minus 6"). This method is neatly illustrated at the bottom of page 112 in Primary Math 3A.

   a) Draw a similar rectangular array that illustrates this method for finding 9×4.

   b) Use this method to mentally multiply the following numbers by 9: 5, 7, 8, 9, 21, 33, and 89.

   c) By this method 7×9 is $70 - 7$. That is less than 70 and more than $70 - 10 = 60$, so its tens digit must be 6. In fact, whenever a 1-digit number is multiplied by 9, the tens digit of the product is _____ less than the given number. Furthermore, the ones digit of $7 \times 9 = 70 - 7$ is $10 - 7 = 3$, the tens complement of 7. When a 1-digit number is multiplied by 9, is the ones digit of the product always the tens complement of the number?

   d) Use the facts of part c) to explain why the "fingers method" (Primary Math 3A page 129) works.

   e) These mental math methods can be used in the course of solving word problems. Answer Problems 5 and 6 on page 131 of Primary Math 3A.

8. *(Mental Math)* Explain how to compute the following mentally by writing down the intermediate step(s) as in Example 5.3.

(a) $5 \times 87 \times 2$
(b) $4 \times 13 \times 25$
(c) 16×11
(d) 17×30.

9. Try to solve the following multi-step word problems in your head.

   • After giving 157¢ to each of 3 boys and 54¢ to a fourth boy, Mr. Green had 15¢ left. How much did he have to start with?

   • After giving 7 candies to each of 3 boys and 4 candies to a fourth boy, Mr. Green had 15 candies left. How many candies did he have at first?

These two problems are solved by the same strategy, but the first is much harder because the first step overloads working memory — while doing the multiplication one forgets the rest of the problem.

a) How would the second problem appear to a student who does not know what 7×3 is? Is there an advantage to instantly knowing $7 \times 3 = 21$, or is it enough for the student to know a way of finding 7×3?

b) If one first observes that $150 \times 3 = (15 \times 3)$ tens $=$ 450, what must be added to solve the first problem? Write down the intermediate steps as in Example 5.3.

1.6 Division

Homework Set 6

1. Identify whether the following problems are using measurement (MD) or partitive division (PD) (if in doubt, try drawing a bar diagram).

   a) Jim tied 30 sticks into 3 equal bundles. How many sticks were in each bundle?

   b) 24 balls are packed into boxes of 6. How many boxes are there?

   c) Mr. Lin tied 195 books into bundles of 5 each. How many bundles were there?

   d) 6 children shared 84 balloons equally. How many balloons did each child get?

   e) Jill bought 8 m of cloth for $96. Find the cost of 1 m of cloth.

   f) We drove 1280 miles from Michigan to Florida in 4 days. What was our average distance per day?

2. To understand the different uses of division, students must see a mix of partitive and measurement division word problems. This problem shows how that is done in the Primary Math books, first in grade 3, then again (with larger numbers) in grade 4.

   Identify whether the following problems use measurement or partitive division by writing MD or PD for each, separated by commas.

   a) Problems 20ef on page 76 of Primary Math 3A.

   b) Problems 4–6 on page 103 and Problems 10 and 11 on page 99 of Primary Math 3A.

   c) Problems 7 – 9 on page 67 of Primary Math 4A.

3. Illustrate with a bar diagram.

   a) measurement division for $56 \div 8$.
   b) partitive division for $132 \div 4$.
   c) measurement division for $2000 \div 250$.
   d) partitive division for $256 \div 8$.
   e) measurement division for $140 \div 20$.
   f) measurement division for $143 \div 21$.

4. Make up a word problem for the following using the procedure of Example 1.6.

   a) measurement division for $84 \div 21$.

   b) partitive division for $91 \div 5$.

   c) measurement division for $143 \div 21$.

5. Illustrate the Quotient–Remainder Theorem as specified.

   a) A number line picture for $59 \div 10$ (show jumps of 10).

   b) A set model for $14 \div 4$.

   c) A bar diagram, using measurement division, for $71 \div 16$.

   d) A rectangular array for $28 \div 6$.

6. One might guess that the properties of multiplication also hold for division, in which case we would have:

   a) Commutative: $a \div b = b \div a$.

   b) Associativity: $(a \div b) \div c = a \div (b \div c)$.

   c) Distributivity: $a \div (b + c) = (a \div b) + (a \div c)$.

   whenever a, b, and c are whole numbers. By choosing specific values of the numbers a, b, and c, give examples (other than dividing by zero) showing that each of these three "properties" is *false*.

CHAPTER 2

Mental Math and Word Problems

2.1 Mental Math

No changes to either the text or the homework.

2.2 Word Problems

Corrections to text: On page 49, line 1–4, note that the structure of the books has changed. Word problems in the Standards edition are no longer preceded by a section of calculations. See pages 68–76 and 80–81. Word problems appear on page 76, while the calculations appear on pages 80–81.

Homework Set 8

1. (*Study the Textbook!*) In Primary Math 3A, read pages 62 and 63, noting the illustrations and filling in the answers in the book (but not on your homework sheet). Then read some of the problems in Practice D on page 64. All of these problems require a two-step solution. For example, Problem 1 is solved, and the steps made clear, as follows.

   Step 1: There are 1930 – 859 = 1071 duck eggs,

   and therefore,

   Step 2: 1930 + 1071 = 3001 eggs altogether.

   a) Give similar two-step solutions to Problems 4, 5, and 6.
   b) Draw a bar diagram and give a similar two-step solution to Problem 21 on page 67.

2. (*Study the Textbook!*) On page 76 of Primary Math 3A, read Problems 20cdef and solve each mentally (no need to write your answers). Carefully read the problems on pages 77-79, paying careful attention to how the bar diagrams are drawn.

   For each problem listed below, draw a bar diagram and then solve. Your solutions should look like those on page 78 and 79.

   a) Problems 8, 10 and 12 of Practice A on page 80.
   b) Problems 9 – 12 of Practice B on page 81.
   c) Problem 11 of Practice C on page 92.

3. Continuing in Primary Math 3A,

   a) Give a two-step solution, as you did in Problem 1 above, to Problems 12 and 13 on page 93.
   b) Which of the word problems on pages 106 and 107 are two-step problems? Notice how in Problem 16

the text helps students by asking two separate questions.

4. Draw a bar diagram and solve the following two-step multiplication problems.
   a) Pierre's weight is 90 kg. He is 5 times as heavy as his daughter. Find the total weight of Pierre and his daughter.
   b) Heather weighs 32 kg. Alexi is twice as heavy as Heather. Olga weighs 21 kg less than Alexi. What is Olga's weight?

2.3 The Art of Word Problems

Corrections to text: Change Exercise 3.3 on page 54 to the following:

EXERCISE 3.3. *Read the section "Multiplying and Dividing by 7" on pages 117–121 of Primary Math 3A. Then do word Problems 7–12 on page 122 and Problem 18 on page 139. Notice how multiplication and division are integrated with addition and subtraction, and how the level of the problems moves upward.*

Homework Set 9

(Study the Textbook!) *Below are some tasks to help you study word problems in Primary Math 5A and Workbook 5A.*

1. In the Primary Mathematics curriculum students get a textbook and a workbook for each semester. The material in the textbook is covered in class, and the workbook problems are done as homework. The students own the workbooks and write in them. Leaf through Primary Math Workbook 5A.
   a) How many pages of math homework do fifth grade students do in the first semester?
   b) If the school year is 180 days long, that is an average of roughly _____ pages of homework per day.
2. a) In Primary Math 5A, read pages 38–40. Notice the arrows at the bottom of the page that direct students (and teachers!) to Workbook exercises. Students do those exercises for homework to consolidate the day's lesson.
   b) Try that homework: in Primary Math Workbook 5A, give Teacher's Solutions for Problems 3 and 4 on page 31 and Exercise 6 on pages 32–34.
3. Returning to Primary Math 5A, give Teacher's Solutions for all problems in Practice C on page 41.
4. In Primary Math 5A, give Teacher's Solutions for Problems 26–29 on page 79 and Problem 24 on page 106.
5. In Primary Math 5A, give Teacher's Solutions for Problems 12, 21–23 on pages 131–132.

CHAPTER 3

Algorithms

3.1 The Addition Algorithm

Corrections to text: Change page 61, line -1 to read: "…, as on pages 60, 61, 64, and 67 in Primary Math 3A."

Homework Set 10

1. Compute using the lattice method: a) 315 + 672 b) 483 + 832 c) 356 + 285 + 261.
2. Order these computations from easiest to hardest:

$$\begin{array}{rr} & 39 \\ + & 70 \\ \hline \end{array} \qquad \begin{array}{rr} & 39 \\ + & 71 \\ \hline \end{array} \qquad \begin{array}{rr} & 30 \\ + & 69 \\ \hline \end{array}$$

3. Reread pages 34–40 in Primary Math 3A. Solve Problem 6 on page 48 and Problems 4–7 in Practice A on page 49 by giving a Teacher's Solution using bar diagrams like those on pages 45–47 (use algorithms — not chip models — for the arithmetic!).
4. (*Study the Textbook!*) Complete the following tasks involving Primary Math 3A.
   a) Page 50 shows an addition that involves rebundling hundreds. For Problems 2–9 on pages 51–53, write down in list format which place values are rebundled (ones, tens, or hundreds). Begin with: 2) ones, 3) tens, 4) hundreds, 5) ones, ones, tens, etc. These include examples of every possibility, and build up to the most complicated case after only three pages!
   b) Illustrate Problems 5bd on page 52 using chip models, making your illustrations similar to the one on page 50. *Include the worked-out arithmetic next to your illustration.*
   c) Similarly illustrate Problems 7bdf on page 52.
   d) Similarly illustrate the arithmetic in Problem 9b on page 53. Explain the steps by drawing a "box with arrows" as shown at the side of page 53.
5. Sam, Julie, and Frank each added incorrectly. Explain their mistakes.

Sam:
$$\begin{array}{rr} & 25 \\ + & 89 \\ \hline & 104 \end{array}$$

Julie:
$$\begin{array}{rr} & 4 \\ & 25 \\ + & 89 \\ \hline & 141 \end{array}$$

Frank:
$$\begin{array}{rr} & 25 \\ + & 89 \\ \hline & 1014 \end{array}$$

3.2 The Subtraction Algorithm

Corrections to text: Change page 63, line -5 to read: "Chip models clarify this – see pages 58–59 of Primary Math 3A."

Homework Set 11

1. The number 832 in expanded form is 8 hundreds, 3 tens, and 2 ones. To find 832 – 578,

   however, it is convenient to think of 832 as _____ hundreds, _____ tens, and _____ ones.

2. To find 1221 – 888, one regroups 1221 as _____ hundreds, _____ tens, and _____ ones.

3. Use the fact that 1000 is "9 hundred ninety ten" to explain a quick way of finding 1000 – 318.

4. Order these computations from easiest to hardest:

$$\begin{array}{rr} & 8256 \\ - & 6589 \\ \hline \end{array} \qquad \begin{array}{rr} & 8003 \\ - & 6007 \\ \hline \end{array} \qquad \begin{array}{rr} & 8256 \\ - & 7145 \\ \hline \end{array}$$

5. (*Study the Textbook!*) Carefully read and work out the problem on page 54 of Primary Math 3A. Then work out similar solutions to the following problems.

   a) Illustrate Problem 5b on page 56 using chip models, making your illustrations similar to the one on page 54. *Include the worked-out arithmetic next to your illustration.*

   b) Similarly illustrate Problem 7b on page 56.

   c) Similarly illustrate Problem 12d on page 58. Explain the steps of Problem 12d by drawing a "box with arrows" as shown in the side of page 58.

6. (*Study the Textbook!*) Page 54 of Primary Math 3A shows a subtraction that involves rebundling thousands. For Problems 1–15 on pages 55–59, write down in list format, without explanations, which place values are rebundled (ones, tens, hundreds, or thousands) and which required bundling across a zero. This teaching sequence includes examples of almost every possibility, and builds up to the most complicated case in only 40 problems!

7. In Primary Math 3A, solve Problems 5–8 on page 60, Problems 5–9 on page 61, and Problems 17–20 on page 67 by giving a Teacher's Solution for each. Your solutions should look like those on pages 45–47.

8. Sam, Julie, and Frank each subtracted incorrectly. Explain each mistake.

Sam:
$$\begin{array}{rrrr} & & 6 & 05 \\ - & & 1 & 39 \\ \hline & & 5 & 34 \end{array}$$

Julie:
$$\begin{array}{rrrr} & 5 & 10 & 15 \\ & \not{6} & \not{0} & \not{5} \\ - & 1 & 3 & 9 \\ \hline & 4 & 7 & 6 \end{array}$$

Frank:
$$\begin{array}{rrrr} & 5 & & 15 \\ & \not{6} & 0 & \not{5} \\ - & 1 & 3 & 9 \\ \hline & & & 6 \end{array} \quad \Rightarrow \quad \begin{array}{rrrr} & 4 & 10 & \\ & \not{5} & & 15 \\ & \not{6} & \not{0} & \not{5} \\ - & 1 & 3 & 9 \\ \hline & 3 & 7 & 6 \end{array}$$

9. The boy pictured above Exercise 2.4 finds 15−7 by adding 5 to the tens complement. Explain how that method is equivalent to finding 15 – 7 by "counting on".

3.3 The Multiplication Algorithm

Corrections to text: Change page 67, line 12 to read: "Study pages 82–83 of Primary Math 3A."

Homework Set 12

1. Compute using the lattice method: a) 21×14 b) 57×39 c) 236×382.
2. (*Study the Textbook!*) Read pages 68–79 of Primary Math 3A. Give a Teacher's Solution to Problem 12 on page 80 and Problem 12 on page 81. Model your solutions on those on pages 78 and 79.

   Why is there a multiplication word problem section just prior to the multiplication algorithm section which starts on page 82?
3. (*Study the Textbook!*) Read pages 82–91 of Primary Math 3A. What stage of the multiplication algorithm teaching sequence is being taught? Illustrate Problem 14g on page 89 using the chip model (as on the bottom of page 87). *Include the worked-out column multiplication.*
4. In Primary Math 3A, solve Problems 2, 9, and 10 on pages 84 – 87, and Problems 8, 11, and 12 on page 92.
5. (*Study the Textbook!*) Read pages 68–72 of Primary Math 4A. These pages develop the algorithm for multiplying by 2–digit numbers. The beginning of Stage 2 is multiplying by multiples of 10. Notice the method taught on page 68 for multiplying by a multiple of 10.
   a) Find 27×60 by each of the three methods of Problem 2 on page 69.
   b) Page 70 makes the transition to multiplying by general 2–digit numbers. What arithmetic property of multiplication is the little boy thinking about in the middle of page 70?
   c) Solve Problem 12e by column multiplication, modelling your solution on Problem 6a on page 70.
6. Give Teacher's Solutions to Problems 8 and 9 of Practice C in Unit 2 of Primary Math 4A. (These are great problems!)
7. Illustrate and compute 37×3 and 84×13 as in Example 3.3 in this section.
8. Sam, Julie, and Frank each multiplied incorrectly. Explain each mistake.

Sam:	Julie:	Frank:
32	2	2
× 7	27	37
2114	× 4	× 4
	88	118

3.4 Long Division by 1–digit Numbers

Corrections to text: Change page 71, line -2 to read: "Primary Math 3A pages 94–107 and Primary Math 4A pages 59–67, . . . "

Chnage page 71, line -1 to read: "Primary Math 4B pages 58–67 . . . "
Change page 72, line 2 to read: ". . . 44–48. This section . . . ".
Change page 72, line 15 to read: "For example, on pages 62–64 of . . . ".
Change page 73, line -3 to read: ". . . pages 96–97 in . . . ".
Change page 74, line -16 to read: "Study pages 101–102 in Primary Math 3A. In the illustration on page 101". . . .
Change page 74, line -12 to read: ". . . within the gray rectangles."

Homework Set 13

1. (*Study the Textbook!*) After looking at page 94 of Primary Math 3A, make up a word problem (not the one illustrated!) that can be used to introduce the definitions of quotient and remainder.
2. (*Study the Textbook!*) Draw your own version of Examples 1–4 on pages 95–96 of Primary Math 3A using exactly the same numbers, but illustrating with dimes (white circles) and pennies (shaded circles) instead of stick bundles.
3. (*Study the Textbook!*) Look at the pictures for Problems

4 and 5 on pages 96–97 of Primary Math 3A. Why is it helpful to move to the chip model instead of staying with bundle sticks?

4. For the problem $243 \div 3$, draw the chip model and the 'box with arrows' as on page 101 of Primary Math 3A. Then do $521 \div 3$ as in Problem 1 on page 102.

5. Make up a measurement division word problem for $45 \div 8$ and solve it as in Example 4.6.

6. a) Using the same procedure as in Example 4.7, write down the reasoning involved in finding $17,456 \div 8$. Begin as follows: How many 8 are in 17,456? Well, 2000 eights gets us to 16,000. That leaves 1,456. Now begin again: . . ..

   b) Write down the long division for $17,456 \div 8$.

7. Give Teacher's Solutions for Problems 4–6 on page 103 and 10–11 on page 99 of Primary Math 3A. At this point students have just learned to do long division; these word problems are intended to provide further practice. Thus the computational part of your Teacher's Solution should show a finished long division, without chip models and without breaking the computation into a sequence of steps. Your solutions should resemble the one given for Problem 11a, page 64 of Primary Math 4A.

3.5 Estimation

Corrections to text: Change page 77, line -10 to read: "In Primary Math 3A, read pages 18-19 and do Problems 1, 4, and 6 on pages 20–22."

Change page 77, line -8 to read: "In Primary Math 4A, read pages 22–23 and the box at the top of page 24, and do Problem 7 on page 24."

Change page 78, line 1 to read:"Primary Mathematics 3A, 4A and 5A."

Change page 78, line 15–17. [1]

Change page 78, line 18 to read: "In Exercise 3 of Primary Math Workbook 5A, . . . "

Change page 79, line 1 to read: "In Primary Math 5A, read page 23 and do Problems 6–8 on page 24."

Change page 79, line 3 to read: "Continuing, read page 25 and do Problems 4–6 on page 26."

Homework Set 14

Write down your solutions to estimation by following the same guidelines as you did for writing Mental Math: write down the intermediate steps in a way that makes clear your thinking at each step.

1. *(Study the textbook!)* a) In Primary Math 3A, reread page 19 and Problem 6 on page 22. Then draw similar pictures to illustrate Problems 7d and 7h on page 23. b) On page 20 of Primary Math 4A, what concept is being reinforced in Problems 1–3? c) On page 23 of Primary Math 4A, read Problem 5 and do Problem 6.

2. *(Study the textbook!)* a) Reread page 56 of Primary Math 4A and do Problem 14 on that page. b) In Primary Math 5A, do Problems 6–11 on page 14. Notice the hints from the children in the margin! c) Do Problems 5cdgh and 6cdgh of Exercise 3 on pages 10–11 in Primary Math Workbook 5A.

3. *(Study the textbook!)* Do all problems in Exercise 6 and Exercise 7 on pages 16–19 in Primary Math Workbook 5A.

4. Estimate by giving a range: a) 57×23, b) 167×347 c) $54,827 \times 57$.

5. When one adds a list of numbers, roundoff errors can accumulate. For example, if we estimate 23 + 41 + 54 by rounding to the nearest ten we get 20 + 40 + 50 = 110, whereas the true sum 118 rounded to the nearest ten is 120. Write down two 3–digit numbers for which rounding to the nearest hundred and adding does not give the sum to the nearest hundred.

6. a) How would you give a high estimate, to the nearest hundred, for 1556 – 371? Which number would you round up? round down?

   b) How would you *underestimate* $3462 \div 28$? (Would you round 28 up or down? How about 3462?)

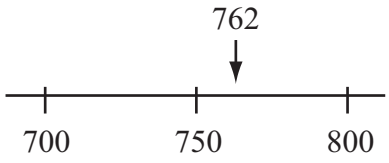
   c) How many 800 pound gorillas can be lifted by an elevator with a capacity of 5750 pounds?

[1] For part a), there are no corresponding problems for Problems 8 and 10 on old page 17. Should page 17 of old 4A be scanned?

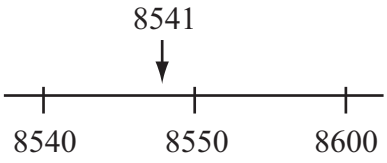
Estimating by rounding to nearest hundred

1. Round to the nearest hundred:

(a) 762 _____



(b) 8541 _____

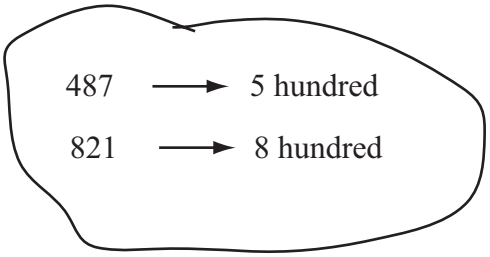


2. Round to the nearest hundred.

(a) 533 (b) 2619 (c) 7564 (d) 8972

3. Round the numbers to the nearest hundred. Then estimate the sum 487 + 821.

5+8 hundred = _____



The sum 487+821 is approximately _____ .

3.6 Completing the Long Division Algorithm

Corrections to text: Change page 83, line 4 to read: "…pages 44–48)."
Change page 83, line 6 to read: "…(See pages 25–26 in Primary Math 5A.)"

Homework Set 15

1. *(Study the textbook!)* Pages 25 – 26 of Primary Math 5A introduce the place value and approximation ideas needed for multi-digit long division. These ideas were briefly described in Section 3.5 as Step 2 of "simple estimation". To see how they are developed for students, carefully read Problem 7 on pages 62 – 63 in Primary Math 4A. Then do the following exercises related to pages 25–27 of Primary Math 5A.
   a) Study page 25. Then draw a similar picture for $3400 \div 100$, putting the student helper's thought bubble *first*. If you were explaining this to a class, would you explain the idea in the thought bubble before or after the chip picture?
   b) Use mental math to find

      $130 \div 10$ $\qquad$ $870 \div 10$ $\qquad$ $4300 \div 100$

      Notice that students who simply "erase all the ending zeros" will obtain three correct answers. Make up a fourth problem of the form ____ $\div 10$ for which "erasing the ending zeros" gives an incorrect answer. (Your problem could be used to assess student understanding).
   c) Do Problem 1 on page 26.
   d) Read Problem 2, paying attention to what the student helper is thinking. This introduces the idea of "cancelling zeros" for divisors which are multiples of 10, 100, and 1000, which effectively allows us to skip the first step of the solution written in color. Write down a similar solution, using two colors and a thought bubble, for $2400 \div 30$.
   e) Do Problem 3.

2. Continuing in Primary Math 5A,
   a) On page 27 do Problems 4a, 5b, and 6adef. For 6d and 6f use (and show) the mental math method of repeatedly dividing by 2.
   b) Returning to page 26, do Problem 4, read Problem 5, and do Problem 6. These are 'simple estimations" done by rounding to a problem like the ones just done.

3. *(Study the textbook!)* Read pages 44–48 of Primary Math 5A.
   a) Do Problems 5adgj on page 45.
   b) What component skill is being emphasized on pages 45–46?
   c) Still on pages 45–46, why are there no chip model pictures?
   d) Do Problems 16abd on page 47.

4. In Primary Math 5A, give Teacher's Solutions for Problems 9, 10, and 13 on page 48. Note that this is a teaching sequence that starts with a 1-step problem and builds up to a multi-step problem.

5. What do you tell Tracy when she writes the following?

```
          7  5    R5
6 | 4  2  3  5
    4  2
    ---------
       0  3  5
       -  3  0
       ---------
             5
```

CHAPTER 4

Prealgebra

4.1 Letters and Expressions

Homework Set 16

Note: Problem 10 requires that you have access to the US edition *of PM 6A.*

1. *(Mental Math)* Make a list of the squares from $11^2 = 121$ through $20^2 = 400$. *Memorize these*. We will use these facts later for Mental Math exercises.
2. *(Mental Math)* Recall that $2^3 = 2 \times 2 \times 2 = 8$. Memorize the "Mental Math tags" $2^5 = 32$, $2^8 = 256$, and $2^{10} = 1024$ and the list of the first 12 powers of 2 (2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096). Using the three tags, you can mentally reconstruct the other powers of 2. For example, 2^7 is $2^5 \times 2 \times 2 = 128$ (two numbers after 32 on the list) and is also $2^8 \div 2 = 128$ (one before 256 on the list). Use the Mental Math tags to find:

   a) 2^4 b) 2^9 c) 2^6 d) 2^{11}

3. Which of the following are algebraic expressions?

   a) $32(52 + 7) - 38 \times \frac{1}{4}$ b) $3 + \div 7$
   c) $a + 3$ d) $5\pi + 4$
   e) $3x + 2 = 7$ f) $(a + b)(a - b)$
   g) x h) $12,304$
   i) $y \div 0$

4. a) Illustrate the expression $a + 7$ using a measurement model.
   b) Illustrate the expression $6x + 2$ using a rectangular array model.

5. Fred is confused about the meaning of the equal sign. His answer to the problem "Simplify $3(x + 2) - x + 8$" is written below. Which of his equal signs are incorrect? What should he have written?

   $$3(x + 2) = 3x + 6 - x = 2x + 6 + 8 = 2x + 14.$$

6. Write the indicated expressions.

   a) The number of inches in m feet.
   b) The perimeter of a square of side s cm.
   c) The value in cents of x nickels and y dimes.
   d) The number of pounds in $6z$ ounces.
   e) Three consecutive whole numbers the smallest of which is n.
   f) The average speed of a train that travels w miles in 5 hours.
   g) Ann is 18 years younger than Bill. Carmen is one-fifth as old as Ann. Dana is 4 years older than Carmen. If Bill is B years old, what is Dana's age in terms of B?

7. Give a Teacher's Solution using algebra (see the template given in this section) for the following problem:

   The lengths of the sides of a triangle, measured in inches, are consecutive whole numbers, and the perimeter is 27 inches. What is the length of the shortest side?

8. You have previously solved the problems on page 41 of Primary Math 5A using bar diagrams. Now do some of those problems again, making the transition to algebra as follows.
   a) For Problem 6, give a Teacher's Solution using a bar diagram.
   b) For Problems 6, 8, and 10 give a Teacher's Solution using algebra only.
9. *(Study the textbook!)* Pages 140–148 of Primary Math 5B introduce students to algebraic expressions.
   a) Do Problems 1 – 14 (beginning on page 141). For each problem write the answer and then write B or E to indicate whether the problem is building an expression or is evaluating one. Your answer for Problem 1 should read 13, $x + 8$; B.
   b) How many different letters are used in the expressions that appear in Problems 1 – 14? Why?
   c) The boy at the bottom of page 142 is calling attention to which arithmetic property?
10. *(Study the textbook!)* In Primary Math 6A:
    a) Illustrate Problems 21ac on page 13 with equations similar to those in the pink boxes on page 12 and Problem 21g with a picture similar to those in Problem 19.
    b) Do Problem 5ab on page 14 by *explicitly* showing the use of the distributive property (see Example 1.14 in this section).
    c) Answer Problems 6–9 on page 14.
11. Make up a short word problem which builds the given expression in the given context. Be sure to make clear what each letter represents.
    a) The expression $12c$ in the context of baking cookies.
    b) The expression $13r + 3s$ in the context of shopping (cf. Exercise 1.9).
    c) The expression $2w+13$ in the context of money saved from an allowance.
    d) The expression $(240 - x)/50$ as the time needed to complete a trip to another city.

4.2 Identities, Properties, Rules

No changes to either the text or the homework.

4.3 Exponents

Homework Set 18

1. *(Mental Math)* Calculate the following mentally, using the Mental Math tags 2^5, 2^8, and 2^{10}.
   a) $32 \cdot 32$ b) $1024 \div 256$
   c) $4096 \div 32$ d) 64×128
   e) $1024 \times 64 \div 512$ f) $2048 \div 256 \times 16$
2. *(Mental Math)* Calculate the following mentally and show how you did it.
   a) $2^8 \cdot 2^7 \div 2^{11}$ b) $(2^3)^5 \div 2^9$
   c) $256 \cdot 128 \div 2048$ d) $8^5 \div 512$
   e) 48×15 f) 256×99
   g) $512\,000 \div 320$ h) 80^3
3. Read page 145 of Primary Math 5B and do Problems 15, 16, and 17ghi (just list the answers separated by commas).
4. *(A Teaching Sequence)* Let m, n and $a \neq 0$ be whole numbers. Simplify the following using the definition of exponents as done in Example 3.3 and Exercise 3.4.
   a) $(5^2)^3$ b) $(5^2)^m$ c) $(a^2)^3$

d) Now simplify $(a^n)^m$, including labels justifying each step, as done in this section for Rules 1, 2, and 4. You may use Rule 1 and "Definition of multiplication" as justifications.

5. *(A Teaching Sequence)* Follow the instructions of Problem 4. Remember to justify Part d)!

a) $3^4 \cdot 5^4$ b) $3^4 \cdot b^4$

c) $3^n \cdot 5^n$ d) $a^n \cdot b^n$

6. Let a and b be non-zero whole numbers. Simplify as much as possible, factoring the numbers and leaving the answer in exponential form.

a) $\dfrac{2^5 \cdot 6^2 \cdot 18^2}{3^4 \cdot 4^2}$ b) $\dfrac{2^5 \cdot (2b)^2 \cdot (2b^2)^2}{b^4 \cdot 4^2}$

c) $\dfrac{a^5 \cdot (ab)^2 \cdot (ab^2)^2}{b^4 \cdot (a^2)^2}$

d) For what values of a and b do your three simplifications become identical?

7. Simplify as in Problem 6 (a, b and c are non-zero).

a) $\dfrac{5^3 \cdot 24^2 \cdot 10^0}{8 \cdot 15^2 \cdot 3}$ b) $\dfrac{a^3 \cdot (bc^3)^2 \cdot (ac)^0}{c^3 \cdot (ab)^2 \cdot b}$

c) How can you obtain simplification a) from b)?

8. Let m and n be two whole numbers. Simplify as in the previous two problems.

a) $\dfrac{6^{21} \cdot 10^{18} \cdot 15^{22}}{30^{11} \cdot 16^7}$ b) $\dfrac{6^{3n} \cdot 10^{n+11} \cdot 15^{22}}{30^{11} \cdot 16^n}$

c) Use the power rules to show that:

$$\frac{6^{3n} \cdot 10^{n+m} \cdot 15^{2m}}{30^m \cdot 16^n} = 3^{3n+m} \cdot 5^{n+2m}.$$

d) Plug $n = 7$ and $m = 11$ into $3^{3n+m} \cdot 5^{n+2m}$. Is the result the same as your answer to a)?

9. *(Calculator)* Describe how to calculate the square of the number $N = 23805723$ using a calculator that displays only 8 digits, plus one pencil-and-paper addition (with possibly more than two summands). *Hint*: Write $N = 2380 \times 10^4 + 5723$ and use the identity for $(a + b)^2$.

10. *(Scientific Notation)* Very large numbers can be conveniently written in the form $c \times 10^N$ where c is a number between 1 and 10 ($1 \le c < 10$) written as a decimal, and N is a whole number. For example, $1,200,000$ is 1.2×10^6. Scientific notation is covered in most middle school curricula.

Write each of the following in scientific notation.

a) The numbers 1030, 15600 and $345,000,000$.

b) The sums $(3.4 \times 10^7) + (5.2 \times 10^7)$ and $(6 \times 10^8) + (9.3 \times 10^8)$.

c) The products $(2 \times 10^4) \times (3.2 \times 10^5)$ and $(8 \times 10^4) \times (96 \times 10^{23})$.

d) The quotients (written in fraction form) $\dfrac{6 \times 10^9}{3 \times 10^4}$ and $\dfrac{5.4 \times 10^8}{9 \times 10^5}$.

e) The powers $(2 \times 10^7)^3$ and $(5 \times 10^4)^3$.

CHAPTER 5

Factors, Primes, and Proofs

5.1 Definitions, Explanations and Proofs

Corrections to text: Change page 110, line 17 to read: "Read Problem 7 on page 34 of Primary Math 4A."

5.2 Divisibility Tests

Homework Set 20

1. Which of the following numbers is divisible by 3? by 9? by 11?

   a) 2, 838 b) 34, 521
   c) 10, 234, 341 d) 792
   e) 8, 394 f) 26, 341
   g) 333, 333 h) 179

2. Which of the numbers below divide the number 5, 192, 132?

   3 4 5 8 9 11

3. Which of the numbers below divide the number 186, 426?

   2 3 4 5 8 9 10 11

4. *(Study the textbook!)* Read all of page 34 of Primary Math 4A. Which Divisibility Tests are described on that page?

5. Let m be a whole number. If 18 divides m, then 3 and 6 divide m as well. Show that the converse is not necessarily true by writing down a number which is divisible by 3 and 6 but not 18.

6. To apply the Divisibility Test for 9 to the 4-digit number 2435, we can write

$$\begin{aligned} 2435 &= 2(1000) + 4(100) + 3(10) + 5 \\ &= 2(999 + 1) + 4(99 + 1) + 3(9 + 1) + 5 \\ &= [2(999) + 4(99) + 3(9)] + [2 + 4 + 3 + 5] \end{aligned}$$

   and note that $[2(999) + 4(99) + 3(9)]$ is a multiple of 9. By the Divisibility Lemma, 2435 is a multiple of 9 if and only if the leftover part $2 + 4 + 3 + 5$ is a multiple of 9, which it is not. Thus 2435 is not a multiple of 9.

a) Similarly show how the Divisibility Test for 3 applies to the number 1134 and the number 53, 648.
b) Similarly show how the Divisibility Test for 11 applies to the number 1358.

7. Prove the Divisibility Test for 9 for 4–digit numbers.

8. Prove the Divisibility Test for 8 (adapt the proof of the Divisibility Test for 4 given in this section).

5.3 Primes and the Fundamental Theorem of Arithmetic

5.4 More on Primes

No changes to either the text or the homework in Sections 5.3 and 5.4.

5.5 Greatest Common Factors and Least Common Multiples

Homework Set 23

1. *(Study the textbook!)* Read pages 26–35 of Primary Math 4A. Notice how the ideas of factors and multiples are introduced, and how common multiples are defined on page 35.
   a) Use the method shown by the little girl in Problem 11 of page 35 to find a common multiple of 15 and 12.
   b) In Practice C on page 36 of Primary Math 4A, do Problems 1 and 4–7.
2. Using only Definition 5.1, prove that $\text{GCF}(a, b) = a$ whenever b is a multiple of a. *Hint:* Why is $\text{GCF}(a, b) \leq a$? Why is $\text{GCF}(a, b) \geq a$?
3. Using only Definition 5.1, prove that if p is prime then $\text{GCF}(p, a) = 1$ unless a is a multiple of p.
4. Use the method of Example 5.4 to find
   a) $\text{GCF}(28, 63)$
   b) $\text{GCF}(104, 132)$
   c) $\text{GCF}(24, 56, 180)$.
5. Use Euclid's Algorithm to find
   a) $\text{GCF}(91, 52)$
   b) $\text{GCF}(812, 336)$, and
   c) $\text{GCF}(2389485, 59675)$.

   Use long division for b) and a calculator for c).
6. Use the method of Example 5.10 to find
   a) $\text{LCM}(32, 1024)$ and b) $\text{LCM}(24, 120, 1056)$.
7. a) On pages 58 and 59 of Primary Math 5A, common multiples are used for what purpose?
   b) Find
   $$\frac{2}{84} + \frac{5}{147} = \_\_\_\_$$
   by converting to fractions whose denominator is $\text{LCM}(84, 147)$.
8. a) Use Euclid's Algorithm to find the GCF of the numbers $2n + 3$ and $n + 1$. *Hint:* Start by writing $2n + 3 = 2(n + 1) + 1$.
   b) Show that the fraction $\dfrac{8n + 1}{20n + 2}$ cannot be reduced for any whole number n.
9. Two gears in a machine are aligned by a mark that is drawn from the center of the first gear to the center of the second gear. If there are 192 teeth on the first gear and 320 teeth on the second gear, how many revolutions of the first gear are needed to realign the mark?
10. The following problem was taken from a fifth grade German textbook. It is for the better students.

    The gymnastics club is having an event, and they want to group all the participants neatly in rows. However, whether they try to use rows of 2, 3, 4, 5, 6, 7 or 8, there is always one gymnast leftover. There are fewer than 1000 gymnasts in all. How many are there?

    Hint: Suppose that one gymnast left the room.

11. a) Write down the prime factorizations of 72 and 112. Then find GCF(112, 72) and LCM(112, 72), and verify that GCF(112, 72) · LCM(112, 72) = 112 · 72.

    b) By referring to the 'prime factorization' methods of finding the GCF and LCM , prove that for any numbers a and b one has

$$\text{GCF}(a, b) \cdot \text{LCM}(a, b) = a \cdot b.$$

    If we know GCF(a, b), this formula can be used to find LCM(a, b)

    c) (*Mental Math*) Find GCF(16, 102) and use the above formula to find LCM(16, 102).

12. Use Euclid's Algorithm to find GCF(57, 23), recording the division facts you use. Then use those division facts to write the GCF as the difference between a multiple of 57 and a multiple of 23 (as explained after Lemma 5.13).

CHAPTER 6

Fractions

6.1 Fraction Basics

Corrections to text: On page 133, line -10, notice that adding fractions is now in Primary Math 3B of the Standards Edition.

Homework Set 24

1. *(Study the textbook!)* [Do this problem only if you have access to Primary Math 3B] Read and work through pages 85–96 of Primary Math 3B. Answer the following problems as you go.
   a) On page 90, draw pictures illustrating Problems 4abc, 5c, 6b, 7b.
   b) On page 94, draw pictures (like those on the same page) for 5e and 5f.
2. *(Study the textbook!)* Read pages 97–100 of Primary Math 3B and 81–86 of Primary Math 4A and answer the following problems.
   a) Parts a, b, c, f, i, and j of Problem 6 on page 84 of Primary Math 4A.
   b) Parts a, b, c, f, i, and j of Problem 12 on page 86 of Primary Math 4A.
   c) Create bar diagrams, similar to those on pages 97–100 of Primary Math 3B and 81–86 of Primary Math 4A, for Problems 4ab and 5ab on page 101 of Primary Math 3B.
3. *(Study the textbook!)* Page 101 in Primary Math 3B and page 87 of Primary Math 4A give some simple fraction word problems.
   Give Teacher's Solutions for Problems 6–9 on page 101 of Primary Math 3B and 6–8 on page 87 of Primary Math 4A. For each subtraction problem, specify whether it is using the part-whole, take-away, or comparison interpretation of subtraction.
4. *(Study the textbook!)* Read Review 3 (pages 106–109) in Primary Math 4A. Review sets like this are designed to evaluate and consolidate student learning.
   a) Answer Problems 13–17 of Review 3 (pages 106–109). For each problem, write the answers as a list of the form 12: $\frac{5}{8}$, $\frac{1}{3}$, ...
   b) Problems 13, 14, 16, and 17 evaluate and consolidate knowledge of what?
5. Using the definition of fractions described in the first few sentences of this chapter, give a "teacher's explanation" (consisting of a number line and one or two sentences) for the equality $\frac{4}{5} = \frac{8}{10}$.

6. Give Teacher's Solutions to the following problems using a picture or diagram based on the indicated model.
   a) Mrs. Smith used $\frac{3}{10}$ of a bottle of cooking oil, which measured 150 $m\ell$. How much oil did the bottle hold? (use an area model).
   b) $\frac{4}{5}$ of the children in a choir are girls. If there are 8 boys, how many children are there altogether? (measurement model).
   c) Jim had 15 stamps. He gave $\frac{2}{5}$ of them to Jill. How many stamps did he give to Jill? (set model).
   d) Beth made 12 bows. She used $\frac{1}{5}$ meter of ribbon for each bow. How much ribbon did she use altogether? (measurement model).
   e) A shopkeeper had 150 kg of rice. He sold $\frac{2}{5}$ of it and packed the remainder equally into 5 bags. Find the weight of rice in each bag. (measurement model).
   f) Peter had 400 stamps. $\frac{5}{8}$ of them were Singapore stamps and the rest were U.S. stamps. He gave $\frac{1}{5}$ of the Singapore stamps to his friend. How many stamps did he have left? (bar diagram).
7. Find a fraction smaller than 1/5. Find another fraction smaller than the one you found. Can you continue this process? Is there a smallest fraction greater than zero? Explain (give an algorithm!).

6.2 More Fraction Basics

Homework Set 25

1. *(Study the textbook!)* Read pages 88–93 of Primary Math 4A, computing mentally as you read. Mixed numbers and improper fractions help students to see that fractions can be bigger than 1.
   a) Illustrate Problems 8a and 8b on page 93 using a number line.
   b) Illustrate Problems 10a and 10b on page 93 using an area model. Start by drawing the whole unit.
2. *(Study the textbook!)* Read pages 54–56 of Primary Math 5A. Do the following problems as you read.
   a) On page 54, Rule 3 is illustrated using which interpretation (partitive or measurement)?
   b) Give Teachers' Solutions to Problems 6–8 of Practice A on page 57.
3. *(Study the textbook!)* Read pages 58–63 of Primary Math 5A. Notice how the problems on pages 58 and 59 teach addition of fractions as a 2-step process (find a common denominator, then add) with the student helpers in the margin finding the least common multiple.
   a) Illustrate Problems 1c, 2c, and 3c of Practice B on page 60 using pictures similar to Examples 2.4 and 2.5 of this section.
   b) Give a Teacher's Solution to Problems 7 and 8 of Practice B on page 60.
   c) Illustrate Problems 1c, 2c, and 4c of Practice C on page 63 using a measurement model similar to Example 2.7 of this section.
4. *(Mental Math)* Do the following Mental Math problems, using compensation for a), b) and c). Show your intermediate steps.
   a) $28\frac{2}{7} - 3\frac{6}{7}$
   b) $9\frac{1}{6} - 5\frac{5}{6}$
   c) $\left(1\frac{3}{4} + 4\frac{5}{11}\right) + \left(2\frac{8}{11} + 5\frac{1}{4}\right)$
   d) $12\frac{1}{8} - 4\frac{5}{8}$
   e) $\frac{4}{9} + \left(\frac{3}{5} + 3\frac{5}{9}\right)$
5. Give range estimates for
   a) $3\frac{8}{9} + 7\frac{3}{13}$, b) $2\frac{2}{11} + 4\frac{8}{9} + 12\frac{1}{12}$.
6. Estimate by rounding to the nearest $\frac{1}{2}$ unit:

a) $2\frac{4}{9} + 7\frac{5}{12}$ b) $22\frac{4}{11} + 19\frac{8}{9} + 13\frac{7}{12}$.

7. Use long division to convert a) $\frac{735}{37}$ and b) $\frac{4271}{9}$ to mixed numbers.
8. A student claims that $\frac{46}{6}$ cannot be equal to $\frac{23}{3}$ because $46 \div 6$ is 7**R**4, while $23 \div 3$ is 7**R**2. How would you respond?
9. a) Use Euclid's Algorithm (cf. Section 5.5) to reduce the fraction $\frac{5829}{18879}$. *Hint*: find GCF(18879, 5829).

   b) Use Euclid's Algorithm to show that the fraction $\frac{13837}{24827}$ cannot be simplified.

6.3 Multiplication of Fractions and a Review of Division

Homework Set 26

1. *(Mental Math)* Another way to write the shortcut for multiplying by 25 is to use fractions:

$$25 \times 48 = \frac{100}{4} \times 48 = 100 \times \frac{48}{4} = 100 \times 12.$$

   Use this way to find the following products mentally.

   a) 25×64 b) 25×320

   c) 884×25 d) 3212×25.

2. *(Mental Math)* Calculate mentally using the arithmetic properties (remember to show your thinking).

   a) $44 \cdot \frac{3}{8} + 44 \cdot \frac{7}{8}$ b) $(\frac{4}{7} + \frac{7}{9}) - \frac{3}{7}$

   c) $48 \times 99\frac{5}{12}$ d) $1234 \cdot \frac{331}{783} + 1234 \cdot \frac{452}{783}$

3. Estimate to the nearest whole number:

   a) $59 \times \frac{1}{3}$ b) $24\frac{1}{4} \times 1\frac{2}{3}$.

4. *(Study the textbook!)* In Primary Math 5A, read pages 67–69, doing the problems in your textbook and studying the 3 methods on page 69. Note how these methods develop the useful principle *cancel first, then multiply*.

   a) Write the answers to Problems 2 and 3 on page 73 as a list of 6 fractions.

   b) Use Method 3 to find $48 \times \frac{23}{12}$ and $320 \times \frac{13}{80}$.

5. *(Study the textbook!)* Read pages 80–82 of Primary Math 5A, noting the area models and studying Method 1 and 2 on page 82. Then answer Problems 5–10 of Practice A on page 83 (no diagrams are necessary).
6. *(Study the workbook!)* Now open Workbook 5A.

   a) Write solutions to Problems 1a and 1b in Exercise 2 on page 81 by drawing an appropriate area model.

   b) Give Teacher's Solutions to Problems 2, 3, and 4 of Exercise 1 on page 80. Use area models for your illustrations.

7. Find the following products by drawing area models like those used in Section 4.2. *Hint:* Start with a rectangle representing a whole unit, then one representing $2\frac{1}{3}$.

   a) $2\frac{1}{3} \times 6$ b) $2\frac{1}{3} \times 6\frac{1}{2}$

   c) $2\frac{7}{8} \times 2\frac{3}{4}$.

8. Show that the distributive property holds for fractions by drawing a picture illustrating that

$$\frac{2}{3}\left(\frac{1}{2} + \frac{1}{3}\right) = \left(\frac{2}{3} \times \frac{1}{2}\right) + \left(\frac{2}{3} \times \frac{1}{3}\right).$$

9. Read the paragraph 'Anticipating, detecting and correcting errors' in the Preface of this book. One common student error is to write

$$2\frac{1}{4} \times 1\frac{1}{3} = 2\frac{1}{12}.$$

   Give an area model and brief explanation which simultaneously shows both the error this student is making and what the correct solution is.

10. *(Study the textbook!)* Read and think about the problems on pages 74–75 and 84–86 of Primary Math 5A. Give Teacher's Solutions for Exercise 3 and Exercise 4 on pages 83–86 in Primary Math Workbook 5A by drawing the bar diagrams like on page 83.
11. Identify whether the following problems are using measurement division (MD) or partitive division (PD). (If in doubt, draw a picture!)

a) If it takes a half-yard of material to make an apron, how many aprons can be made with 3 yards of material?

b) How many half bushels are there in $2\frac{1}{4}$ bushels?

c) The perimeter of a square flower bed is 32 feet. Find the length of each side.

d) Mary poured 6 cups of juice equally into 8 glasses. How much was in each glass?

e) How many laps around a 1/4 mile track make 6 miles?

f) We drove 3240 miles from New York to Los Angles in 6 days. What was our average distance per day?

6.4 Division of Fractions

Homework Set 27

1. *(Study the textbook!)* Read pages 88 and 89 in Primary Math 5A. Give Teacher's Solutions to Problems 4–10 in Practice C on page 90.
2. Illustrate the following with a bar diagram and solve the problem.
   a) measurement division for $2 \div \frac{1}{3}$.
   b) measurement division for $\frac{3}{2} \div \frac{1}{6}$.
   c) partitive division for $14 \div 4$.
   d) partitive division for $\frac{2}{5} \div \frac{1}{3}$.
   e) measurement division for $\frac{7}{2} \div \frac{1}{4}$.
   f) partitive division for $5 \div \frac{7}{3}$.
   g) partitive division for $\frac{2}{3} \div 3$.
   h) partitive division for $\frac{5}{3} \div \frac{8}{3}$.
3. Give a Teacher's Solution like the first solution to Example 4.8:
   a) After spending $\frac{4}{7}$ of her money on a jacket, Rita had \$36 left. How much money did she have at first?
   b) While filling her backyard swimming pool, Anita watched the level rise from $\frac{1}{9}$ full to $\frac{1}{3}$ full in $\frac{2}{3}$ hour. What is the total time required to fill the pool?
4. Give a Teacher's Solution like the second solution to Example 4.8:
   a) After reading 186 pages, Jennifer had read $\frac{3}{5}$ of her book. How many pages long was the book?
   b) A dump truck contains $\frac{2}{3}$ of a ton of dirt, but is only $\frac{3}{10}$ full. How many tons of dirt can the truck hold?
5. Give a Teacher's Solution using algebra:
   a) $\frac{3}{7}$ of the coins in a box are nickels. The rest are pennies. If there are 48 pennies, how many coins are there altogether?
   b) A farmer took $\frac{3}{4}$ hour to plow $\frac{2}{5}$ of his corn field. At that rate, how many hours will be needed to plow the entire field?
6. Give a Teacher's Solution to each of the following problems.
   a) Michelle spent $\frac{3}{5}$ of her money on a backpack. With the rest of her money she bought 3 CDs at \$12 each. How much did the backpack cost?
   b) Whitney made a large batch of cookies. She sold $\frac{2}{3}$ of them and gave $\frac{1}{5}$ of the remainder to her friends. If she had 60 cookies left, how many cookies did she originally make?
   c) Tony spent $\frac{2}{5}$ of his money on a pair of running shoes. He also bought a coat which cost \$6 less than the shoes. He then had \$37 left. How much money did he have at first?
   d) A fish tank weighs 11.5 lbs when it is $\frac{1}{8}$ full of water and 34 lbs when it is $\frac{3}{4}$ full. How much does the empty tank weigh?

6.5 Division Word Problems

6.6 Fractions as a Step Toward Algebra

No changes to either the text or the homework in Sections 6.5 and 6.6.

CHAPTER 7

Ratios, Percentages, and Rates

7.1 Ratios and Proportions

Corrections to text: Change page 168, line 4 to read: "…pages 135–138 of …".
Change page 168, line -10 to read: "Read pages 135–140 of …".
Change page 168, line -8 to read: "On page 139, how …".
Change page 168, line -6 to read: "On page 140, the …".
On page 171, lines 1–2 notice that the algebraic approach to ratios is introduced in the sixth grade.

Homework Set 30

Note: Problem 5 requires that you have access to the US edition *of PM 6A.*

1. *(Study the textbook!)* Read pages 135–142 of Primary Math 5A, doing the problems mentally as you read. In Practice A on page 143, answer Problems 1–4 and give Teacher's Solutions for Problems 5–7.
2. *(Study the textbook!)* Read pages 143–145 of Primary Math 5A, doing the problems mentally as you read. In Practice B on page 146, answer Problems 4–7 and give Teacher's Solutions for Problems 8 and 9.
3. Draw a picture illustrating why the ratios 12:16, 6:8 and 3:4 are equal (see page 139 of Primary Math 5A).
4. In Workbook 5A, read and answer problems on pages 129–137 by filling in the answers in the workbook (do not copy onto your homework paper).
5. *(Study the textbook!)* Read pages 21–33 of Primary Math 6A, doing the problems mentally as you read. Give Teacher's Solutions for Problems 6–8 of Practice 3A and Problems 6–9 of Practice 3B.
6. If $a : b = c : d$, prove that a) $\frac{a}{b} = \frac{c}{d}$ and b) $ad = bc$.
   Hint: By Definition 1.4, if the ratios $a : b$ and $c : d$ are equivalent then there is a (nonzero) number x so that $a = cx$ and $b = dx$.
7. Reread the three student solutions in Example 1.14 above. Who is right, and what did the other two do incorrectly? (*Hint:* it will help to be very clear about units. Diana's ratio 3 : 5 is a count using what unit? Her ratio 1 : 5 is a count using what unit? Kevin's $\frac{3}{5}$ means $\frac{3}{5}$ of what unit?)
8. A class is presented with the following problem.
   In a bag of marbles, the ratio of white marbles to red marbles is 2 : 3 *and the ratio of red to black marbles is* 6 : 11. *What is the ratio of white to black marbles?*

Conner, who likes using fractions, writes $\frac{2}{3} \times \frac{6}{11} = \frac{4}{11}$ and announces, without explanation, that the ratio of white to black marbles is 4 : 11. Explain why he is right. (*Hint:* taking the number of black marbles to be the whole unit, what fraction are red, and then what fraction are white?)

7.2 Changing Ratios and Percentages

Homework Set 31

Note: Problems 1, 2, 3, 7, and 8 require that you have access to the US edition *of PM 6A.*

1. *(Study the textbook!)* Read pages 34–38 of Primary Math 6A doing the problems mentally as you read. Give Teacher's Solutions for Problems 1, 2, 4, 6, 7, 8 in Practice **3C**.
2. Give Teacher's Solutions for Problems 26–29 of Review A in Primary Math 6A.
3. Give Teacher's Solutions for Problems 26–31 of Review B in Primary Math 6A.
4. Give a Teacher's Solution *using algebra* for the problem solved in Example 2.3 of this section. *Hint:* For a) Let x be the number of dollars each had at first (so x corresponds to the '?' in the bar diagram).
5. Give a Teacher's Solution using algebra for the problem solved in Example 2.4 of this section.
6. Give a Teacher's Solution *using algebra* for the following problem: A truck contains 1000 pounds of sand and concrete in the ratio 2 : 3. After x pounds of sand is added, the ratio of the sand and concrete becomes 4 : 5. Find x.
7. *(Study the textbook!)* Read pages 47–52 of Primary Math 6A, doing the problems mentally as you read. Answer all problems in Practice 4A. For Problems 6–10, also state what the 'whole unit' is in the problem.
8. *(Study the textbook!)* Read pages 55–59 of Primary Math 6A. Then answer all problems in Practice 4C. For Problems 5–10, state what the 'whole unit' is and give Teacher's Solutions to Problems 9 and 10.

7.3 Solving Percent Problems by the Unitary Method

Homework Set 32

Note: Problems 1, 3, 4, and 5 require that you have access to the US edition *of PM 6A.*

1. In Primary Math 6A, answer Problems 1–4 of Practice 4B. Then give Teacher's Solutions for Problems 5–10 of Practice 4B.
2. Give a Teacher's Solution with algebra for the problem of Example 3.1.
3. *(Study the textbook!)* Read pages 61–66 of Primary Math 6A, doing the problems mentally as you read. Then answer Problems 7–10 in Practice 4D, identifying the whole unit and giving a Teacher's Solution for each.
4. a) Write your solution to Exercise 3.5 in this section.
   b) Give similar solutions to Problems 1, 4, 5, and 6 in Practice 4D of Primary Math 6A.
5. Give clear solutions to each of the problems in Practice 4E in Primary Math 6A. There is no need to include a diagram with each, but each solution should include a few words of explanation to make the reasoning clear. (Problem 6 is tricky; the answer is not 20%).
6. Consider the following problem:

   > In the spring Woody Woodchuck lost 25% of his weight. Then he gained 20% in the summer and lost 10% in the fall. In the winter he again gained 20%. Did he lose or gain weight that year?

   a) Give a clearly written solution starting with "Using

a unit of weight so that Woody's initial weight is 100 units, . . . "

b) Give a clearly written solution starting with "If Woody initially weighed x lbs, then . . . "

7.4 Rates, Speed, and Arithmetic with Units

Homework Set 33

Note: Problems 2, 3, and 10 require that you have access to the US edition *of PM 6A.*

1. **(Do this problem only if you have Primary Math 5B)** Read pages 111–117 of Primary Math 5B calculating each problem mentally as you read. Give Teacher Solution's for the problems in Practice B on page 117.
2. Read pages 74–80 of Primary Math 6A, doing each problem mentally as you read. Answer all problems in Practice 5A.
3. Answer all problems in Practice 5B of Primary Math 6A.
4. Water flows from a hose at a rate of 1.9 liters per second. How many gallons per minute is that? (1 gal $\approx$ 3.8 ℓ).
5. Water evaporates from a pond at a rate of 0.3 liters per square meter per day. If the pond has a surface area of 6000 square meters, how much water evaporates per week?
6. A car travels from City A to City B at an average speed of 60 mph. The car returns via the same route at an average speed of 40 mph. The average speed of the car for the entire trip is:
   a) 52 mph b) 50 mph
   c) 48 mph d) Cannot be determined.
7. One hose can fill a swimming pool in 30 hours, while the other hose fills the swimming pool in 70 hours. How long would it take to fill up the swimming pool using both hoses together?
8. It takes one corn mill 6 minutes to grind a 50 pound bag of corn into cornmeal, while it takes a slower mill 9 minutes to grind a bag of corn. If both mills are working at the same time, how long would it take to grind 1500 pounds of corn?
9. A machine shop polishes small metal parts using three polishing machines. Machine A and Machine B, working together, take $1\frac{1}{2}$ hours to polish a ton of parts. Machine A and Machine C take 1 hour to polish a ton of parts, while Machine B and Machine C require only $\frac{3}{4}$ hour to polish a ton of parts. Find each machine's working rate in tons per hour.

   Hint: Start by defining Machine A's rate to be a ton/hour, Machine B's rate to be b ton/hour, and Machine C's rate to be c ton/hour. Then find a, b, and c.
10. Look through the problems in Review D of Primary Math 6A (pages 83–86). Like all Reviews in the Primary Math books, this is designed to evaluate and consolidate learning. Try part of it yourself. First answer the question: what division is asked for in Problem 10? Then solve Problems 20–37.

CHAPTER 8

Negative Numbers and Integers

8.1 Negative Numbers

8.2 Arithmetic with Integers

8.3 Integers as a Step Towards Algebra

No changes to either the text or the homework in all of Chapter 8.

CHAPTER 9

Decimals, Rational and Real Numbers

9.1 Decimals

9.2 Rational Numbers and Decimals

9.3 Real Numbers and Decimals

9.4 Newton's Method and $\sqrt{2}$

No changes to either the text or the homework in all of Chapter 9.